

Classical solution for ghost D-branes in string field theory

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A ghost D-brane has been proposed as an object that cancels the effects of a D-branes. We construct a classical solution with an arbitrary number of D-branes and ghost D-branes in the context of open string field theory. Cancellation of BRST cohomology between D-branes and ghost D-branes is shown.

1 Introduction

Recent developments in open string field theory (OSFT) [1] have proved that it is able to describe wide variety of open string backgrounds. A prominent example is the solution found by Erler and Maccaferri [2] which covers wide range of open string backgrounds such as marginal deformation, D-brane lump and multiple D-branes. More recently, Kojita, Maccaferri, Masuda and Schnabl have incorporated topological defects between boundary conformal field theories (BCFTs) [3] into OSFT context [4]. A common belief behind those developments is that OSFT governs the landscape of (hopefully all possible) BCFTs. Studies on explicit background are still important to understand such landscape.

In this paper, we are interested in *ghost D-branes*, which are rather different from D-branes in conventional BCFTs. Originally, ghost D-branes (gD-branes) were introduced by Okuda and Takayanagi as objects that cancel the effect of D-branes [5] and studied subsequently by some authors [6, 7]¹. It is characterized by a boundary state with opposite sign:

$$|B(gD)\rangle = -|B(D)\rangle. \quad (1)$$

Then, the amplitude for closed string propagation between D and gD branes is given by $\langle B(gD)|\Delta|B(D)\rangle = -\langle B(D)|\Delta|B(D)\rangle$, where Δ is an inverse of the closed string

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¹ The author became aware of works [8, 9, 10, 11, 12, 13] which also deal with branes with negative tension. We thank S. E. Parkhomenko for correspondence.

propagator. Obviously, this has a sign opposite to the amplitude with two D-branes. In open string channel, these amplitudes are interpreted as one-loop partition functions of open strings stretched between branes. The negative sign of D-gD partition function can be attributed to fermionic path integral. Thus, field content on a coincident D-gD pair is

$$\begin{pmatrix} \varphi_1 & \psi \\ \psi^\dagger & \varphi_2 \end{pmatrix} \quad (2)$$

where φ_1 and φ_2 are bosonic while ψ and ψ^\dagger are fermionic. Extension to N D-branes and M gD-branes give rise to $U(N|M)$ or $OSp(N|M)$ matrix with similar structure as (2). The authors of [5] showed that D and gD branes cancel each other in the partition function of supermatrix model defined by (2). With that result, they claimed that a gD-brane cancels D-brane completely therefore a D-gD pair is equivalent to the tachyon (or closed) string vacuum. This means that all physical observables cancel between D and gD branes.

Subsequently, several authors encountered gD-branes in the efforts of constructing solution for multiple D-branes in OSFT [14, 15, 16, 17, 18, 19]². In this context, a gD-brane is given by a classical solution with negative tension. It has not been drawn much attentions because of its peculiar nature. However, it is worthwhile to mention that single gD-brane was found to be regular solution in all known literature. In particular, the gD-brane solution discovered by Masuda, Noumi and Takahashi (MNT) [17] passed thorough most stringent consistency checks ever known. On the other hand, a solution for multiple D-branes in universal space, which was main aim of the authors of [14, 15, 16, 17, 18, 19], turned out to fail such consistency checks unless quite subtle regularization (for example, phantom term) is assumed. Therefore, it is worth to investigate gD-branes seriously in the context of OSFT.

In this paper, we will ask two questions about nature of gD-brane. First, we will ask whether it is *physical degrees of freedom*. Its negative tension implies inconsistency of bosonic OSFT. Therefore it is important to ask whether there exists a logic to exclude gD-brane from the spectrum. Unfortunately, we will not have conclusive answer to this question. We will discuss about possible solutions of the problem in terms of gauge fixing.

Second, we will ask *how gD-branes in OSFT differ from the original picture of Okuda and Takayanagi*. A crucial difference between them is seen in a way to cancel D and gD branes. In the original picture, the cancellation completely holds at quantum level since partition function becomes trivial for D-gD pairs. On the other hand, the cancellation in OSFT was confirmed only for physical observables. To close the gap between two, we will propose another criterion for the cancellation. Namely, it is cancellation of *BRST cohomology*. It will be shown that the BRST cohomology of D-gD pairs vanish with suitable choice of the string field.

Before going into details, we briefly sketch main results of our paper. MNT gD-

²Refs. [20, 21] also rediscovered gD-branes as boundary states.

brane [17] is simply given by a sum of two “tachyons”:

$$\Psi_{MNT} = \Psi_F + \Psi_H, \quad (3)$$

where Ψ_F and Ψ_H are Okawa-type analytic solutions [22] that are orthogonal with each other:

$$\Psi_F \Psi_H = \Psi_H \Psi_F = 0. \quad (4)$$

In order to prove D-gD cancellation of cohomology, we fix Ψ_F to be tachyon and switch to the vacuum defined by

$$Q_F = Q_B + \{\Psi_F, *\}. \quad (5)$$

Then, a solution for a D-gD pair is identified as

$$\Psi_{D+gD} = -\Psi_F + \Psi_H. \quad (6)$$

BRST cohomology around this solution vanishes as we will see, since

$$\begin{aligned} Q_{D+gD} &= Q_F + \{\Psi_{D+gD}, *\} \\ &= Q_B + \{\Psi_H, *\} \end{aligned} \quad (7)$$

just corresponds to a kinetic operator for “another” tachyon. Thus, we have shown that a D-gD pair Ψ_{D+gD} has no cohomology therefore physical excitations cancel out. We also construct a solution with an arbitrary numbers of D and gD branes:

$$\Psi_{N,M} = - \sum_{a=1}^N \Sigma_{k_a} \Psi_F \bar{\Sigma}_{k_a} + \sum_{a=1}^M \Sigma_{l_a} \Psi_H \bar{\Sigma}_{l_a}, \quad (8)$$

where Σ_{k_a} and $\bar{\Sigma}_{k_a}$ are the modified BCC projectors introduced in [2]. It will be shown that the cohomology not always cancels but cancels in a subset of whole string fields.

This paper is organized as follows. In section 2, after a brief review of the MNT solution [17], we will introduce a set of solutions which describes the D-gD system. Cancellation between D and gD branes will be confirmed for gauge invariant observables and BRST cohomology. In section 3, we will extend this result to multiple branes. We will conclude in section 4 with some discussions.

2 MNT ghost D-brane

2.1 Solutions

We begin with a study of the ghost D-brane solution discovered by Masuda, Noumi and Takahashi (MNT) [17]. As explained in introduction, it is simply a sum of formal solutions [22] of equation of motion around the perturbative vacuum:

$$\Phi_{MNT} = \Psi_F + \Psi_H, \quad (9)$$

where

$$\Psi_F = Fc \frac{K}{1-F^2} BcF, \quad \Psi_H = Hc \frac{K}{1-H^2} BcH, \quad (10)$$

where c and B are elements of the KBc algebra, and F and H are functions of K . Since the equation of motion is quadratic, a sum of two Okawa solutions never becomes a solution unless their product vanish, i.e.,

$$\Psi_F \Psi_H = \Psi_H \Psi_F = 0. \quad (11)$$

We will refer this relation as *orthogonality*. Two solutions (10) becomes orthogonal when $FH = 1$ holds. It is also required that both Ψ_F and $\Psi_H = \Psi_{F^{-1}}$ derive the expected values of the classical action (in this paper, normalized to be -1) and vanishing cohomologies. In addition, the authors impose a consistency condition for the boundary state [23] to the solutions. They showed an explicit choice

$$F = \sqrt{\frac{1-pK}{1-qK}}, \quad H = \sqrt{\frac{1-qK}{1-pK}}, \quad (12)$$

where p and q are positive numbers, fulfills all requirements. With this choice, they showed that the solution (9) should be identified with a gD-brane. In order to confirm this, it is convenient to switch to the tachyon vacuum. We fix Ψ_F to be the “reference” tachyon vacuum³ so that the theory is described by an action

$$S_F[\Psi] = \text{Tr} \left[\frac{1}{2} \Psi Q_F \Psi + \frac{1}{3} \Psi^3 \right] \quad (13)$$

where $Q_F = Q_B + \{\Psi_F, *\}$. Then, equation of motion of the action (13) is $Q_F \Psi + \Psi^2 = 0$ and we find four solutions composed by Ψ_F and Ψ_H :

$$\Psi_D = -\Psi_F, \quad (14)$$

$$\Psi_{TV} = 0, \quad (15)$$

$$\Psi_{D+gD} = -\Psi_F + \Psi_H \quad (16)$$

$$\Psi_{gD} = \Psi_H. \quad (17)$$

It is straightforward to confirm that all of them satisfy the equation of motion. We identify Ψ_D , Ψ_{TV} , Ψ_{D+gD} and Ψ_{gD} as D-brane, tachyon vacuum, D-gD pair and gD-brane respectively. An evidence of this identification comes from values of tension; 1 for D-brane, 0 for the tachyon vacuum and D-gD pair and -1 for gD-brane.

2.2 Gauge invariant observables

In this section, we evaluate three gauge invariant observables: classical action, gauge invariant overlap and boundary state. Let $\mathcal{O}(\Psi)$ be a gauge invariant observables. Then the cancellation is represented by

$$\mathcal{O}(\Psi_{D+gD}) = 0, \quad (18)$$

³ As is clear from (12), there is no essential difference between F and H since they exchange under $p \leftrightarrow q$. Ψ_F and Ψ_H are gauge equivalent since they derive same physical observables therefore belong to same class of analytic solutions [17].

where \mathcal{O} is an observable of interest.

The cancellation of the tension is easily established as follows:

$$\begin{aligned} S_F[\Psi_{D+gD}] &= S_F[\Psi_D] + S_F[\Psi_{gD}] \\ &= 1 + (-1) \\ &= 0, \end{aligned} \tag{19}$$

where we have used orthogonality in the first line.

We also confirm cancellation of gauge invariant overlap for closed string [24, 25]. The evaluation goes straightforwardly as

$$\begin{aligned} \langle I | \mathcal{V}(i) | \Psi_{D+gD} \rangle &= \langle I | \mathcal{V}(i) | \Psi_D \rangle + \langle I | \mathcal{V}(i) | \Psi_{gD} \rangle \\ &= -\langle I | \mathcal{V}(i) | \Psi_F \rangle + \langle I | \mathcal{V}(i) | \Psi_H \rangle \\ &= 0, \end{aligned} \tag{20}$$

where \mathcal{V} is a closed string vertex operator and $\langle I |$ is the identity string field. In third line, we used the fact that the values of the overlap for are common for tachyons. Above result (20) can be further extended to cancellation of boundary state with a help of Ellwood conjecture [24]:

$$\langle \mathcal{V} | c_0^- | B(\Psi) \rangle - \langle \mathcal{V} | c_0^- | B(\Psi_0) \rangle = \langle I | \mathcal{V}(i) | \Psi \rangle, \tag{21}$$

where Ψ_0 refers to a classical solution for the reference BCFT⁴. Similarly, Ψ refers to an open string field in the reference OSFT at the perturbative vacuum. Applying (20) to above equation we have

$$\begin{aligned} \langle \mathcal{V} | c_0^- | B(\Psi_0 + \Psi_{D+gD}) \rangle - \langle \mathcal{V} | c_0^- | B(\Psi_0) \rangle &= \langle I | \mathcal{V}(i) | \Psi_{D+gD} + \Psi_0 \rangle \\ &= 0, \end{aligned} \tag{22}$$

where we have set Ψ_0 as zero. We regard boundary states in left hand side of above equation as the gauge invariant boundary state introduced by Kudrna, Maccaferri and Schnabl with assuming the uplift to the auxiliary CFT [26]. Since their boundary states is linear with respect to open string fields, (22) is reduced to

$$\langle \mathcal{V} | c_0^- | B(\Psi_{D+gD}) \rangle = 0. \tag{23}$$

As explained in [26], Ellwood conjecture (21) essentially contains all information of closed string state therefore can be extended to arbitrary closed string state. Therefore we have

$$|B(\Psi_{D+gD})\rangle = 0, \tag{24}$$

which can be interpreted as cancellation of boundary state⁵.

⁴We omit $4\pi i$ factor in right hand side of (21).

⁵MNT confirmed the cancellation in boundary state in rather different way. They used KOZ boundary state [23] which is neither linear nor gauge invariant. They showed directory $|B(\Psi_D)\rangle = |B\rangle$ and $|B(\Psi_{gD})\rangle = -|B\rangle$.

2.3 Cohomology

As explained in introduction, it is easy to find empty cohomology of a D-gD pair. Namely, we evaluate the kinetic operator as

$$\begin{aligned}
Q_{D+gD} &= Q_F + \{\Psi_{D+gD}, *\} \\
&= Q_B + \{\Psi_F, *\} + \{-\Psi_F + \Psi_H, *\} \\
&= Q_B + \{\Psi_H, *\} \\
&= Q_H.
\end{aligned} \tag{25}$$

This is nothing but a BRST charge shifted by “another” tachyon vacuum specified Ψ_H therefore has a nontrivial homotopy operator [27]

$$A = \frac{1 - H^2}{K} B, \tag{26}$$

and of course, its cohomology vanishes. This means that there are no physical excitations around a D-gD pair even at quantum level. An OSFT around a D-gD pair is completely equivalent to that around the tachyon vacuum since their cohomologies are identical. Thus, our result validate the statement *A system with a pair of D-brane and ghost D-brane located at the same location is physically equivalent to the closed string vacuum* [5] in the context of OSFT.

2.4 Projections

Our construction of D-gD system is largely owing to the orthogonality between Ψ_F and Ψ_H . The identity $\{\Psi_F, \Psi_H\} = 0$ means that Ψ_H belongs to the kernel of the background shift generated by $\{\Psi_F, *\}$. It can be understood that such string fields are not limited to Ψ_H but fill large part of the space of string fields. A crucial observation is that both Ψ_F and Ψ_H are projected string fields:

$$\Psi_F = p_1 \Psi_F q_2, \quad \Psi_H = p_2 \Psi_H q_1, \tag{27}$$

where p_i and q_i ($i, j = 1, 2$) are star algebra projectors defined by

$$p_1 = FcBH, \quad q_1 = FBcH, \tag{28}$$

$$p_2 = HcBF, \quad q_2 = HBcF. \tag{29}$$

p_i and q_i are orthogonal projectors since $p_i + q_i = 1$. Projectors with different indexes do not always commute but obey rather non-trivial algebra which is summarized as

$$p_i p_j = p_i, \quad q_i q_j = q_j, \tag{30}$$

$$p_i q_j = 0, \quad q_i p_j = \epsilon_{ij}(p_j - p_i), \tag{31}$$

where ϵ_{ij} is the antisymmetric tensor. Using these projectors, one can decompose arbitrary string field into projected sectors⁶. Here we are interested in two kinds of decompositions. One is “D-like” decomposition which is given by

$$\begin{aligned}\Psi &= (p_1 + q_1)\Psi(p_2 + q_2) \\ &= p_1\Psi p_2 + p_1\Psi q_2 + q_1\Psi p_2 + q_1\Psi p_2 \\ &= \psi_1 + \psi_2 + \psi_3 + \psi_4,\end{aligned}\tag{32}$$

and the other is “gD-like” decomposition

$$\begin{aligned}\Psi &= (p_2 + q_2)\Psi(p_1 + q_1) \\ &= p_2\Psi p_1 + p_2\Psi q_1 + q_2\Psi p_1 + q_2\Psi p_1 \\ &= \phi_1 + \phi_2 + \phi_3 + \phi_4.\end{aligned}\tag{33}$$

One can readily find that $\Psi_D = -\Psi_F$ and $\Psi_{gD} = \Psi_H$ are components of ψ_2 and ϕ_2 respectively. Then, the orthogonality between Ψ_F and Ψ_H is not limited to these solutions but can be extended to the projected sectors:

$$\psi_2\phi_2 = \phi_2\psi_2 = 0.\tag{34}$$

Usually, one may implicitly assume that a nontrivial solution of equation of motion causes background shift for arbitrary string field. However, it is not true if that solution has a kernel. For illustration, we consider OSFT action around the tachyon vacuum with expanding the fluctuation in gD-like expansion (33):

$$\Psi = \Psi_F + \phi_2 + \eta,\tag{35}$$

where $\eta = \phi_1 + \phi_3 + \phi_4$. In this case, the string field ϕ_2 is the kernel of the solution Ψ_F . The OSFT action is expanded as

$$S[\Psi] = \frac{1}{2}\text{Tr}[\eta Q_F \eta] + \frac{1}{2}\text{Tr}[\phi_2 Q_B \phi_2] + \text{Tr}[\phi_2 Q_B \eta] + (\text{cubic}) + (\text{const.}).\tag{36}$$

It is observed that kinetic terms with ϕ_2 are not shifted to that for the tachyon vacuum (Q_F) but remain un-shifted (Q_B). Therefore, ϕ_2 can be interpreted as a degrees of freedom on a “residual” D-brane. Equations of motion for the action (36) are

$$Q_B\phi_2 + \phi_2^2 + Q_B\eta + \phi_2\eta + \eta^2 = 0,\tag{37}$$

$$Q_F\eta + \eta^2 + Q_B\phi_2 + \phi_2^2 + \phi_2\eta = 0.\tag{38}$$

Since ϕ_2 and η are projected components, one can project out either of them. By setting $\eta = 0$, both of the above equations reduce to the equation of motion for a D-brane

$$Q_B\phi_2 + \phi_2^2 = 0.\tag{39}$$

⁶We are inspired by a work [28] in which the string field is decomposed by the KMTT projectors.

Thus tachyon condensation takes place again with a solution $\phi_2 = \Psi_H$. The negative tension of the solution can be interpreted as a result of tachyon condensation in the residual sector. Thus existence of the residual sector is responsible to the peculiar nature of gD-brane which has negative tension. One can argue whether the residual sector can be removed by gauge fixing. A condition that picks up η component from (33) reads

$$p_2 \Psi q_1 = 0. \quad (40)$$

This condition looks like a sort of linear b gauge fixing [29] since p_2 and q_1 include products of B and c . In order to check validity of the condition, one has to show that arbitrary string field can be gauge transformed to satisfy (40), and also that no residual gauge symmetry is left. Proof seems to require detailed and careful evaluation of gauge transformation, so we leave it as a future task.

3 Multiple branes

We next turn to construction of a solution with arbitrary number of D and gD branes. Basic ingredients of our construction is the modified boundary condition changing (BCC) operators [2]

$$\Sigma_a = Q_F (FB\sigma_a F), \quad \bar{\Sigma}_a = Q_F (FB\bar{\sigma}_a F), \quad (41)$$

where σ_a and $\bar{\sigma}$ are BCC operators associated with certain boundary conditions, and a corresponds to Chan-Paton factor which labels D or gD brane. By construction, they are Q_F exact therefore vanish when multiplied with it:

$$Q_F \Sigma_a = Q_F \bar{\Sigma}_a = 0. \quad (42)$$

They also inherit the algebra of original BCC operators:

$$\bar{\Sigma}_a \Sigma_b = \delta_{ab}, \quad \bar{\Sigma}_a \Sigma_b = \mathbf{finite} \times \delta_{ab} \quad (43)$$

We consider the theory at the tachyon vacuum whose kinetic operator is Q_F . Then, it is easily understood that, if Φ is a solution of the equation of motion, $\Sigma_a \Phi \bar{\Sigma}_a$ is also a solution. Therefore, for given set of solutions $\Phi_1, \Phi_2, \dots, \Phi_N$, we can construct a set of mutually orthogonal solutions

$$\Sigma_1 \Phi_1 \bar{\Sigma}_1, \Sigma_2 \Phi_2 \bar{\Sigma}_2, \dots, \Sigma_N \Phi_N \bar{\Sigma}_N. \quad (44)$$

Of course, their sum is also a solution due to the orthogonality of the modified BCC projectors. The sum is conveniently described by vector notation:

$$(\Phi_1, \Phi_2, \dots, \Phi_N) = \sum_{a=1}^N \Sigma_a \Phi_a \bar{\Sigma}_a \quad (45)$$

We can easily construct solutions for multiple D or gD branes respectively by

$$\Psi_D^{(N)} = \underbrace{(\Psi_D, \Psi_D, \dots, \Psi_D)}_N, \quad (46)$$

$$\Psi_{gD}^{(N)} = \underbrace{(\Psi_{gD}, \Psi_{gD}, \dots, \Psi_{gD})}_N, \quad (47)$$

where Ψ_D and Ψ_{gD} are brane solutions defined by (14) and (17) respectively. Due to the orthogonality, gauge invariant observables split into pieces for each component. For example, classical action for D-branes is evaluated as

$$S_F[\Psi_D^{(N)}] = N S_F[\Psi_D] \quad (48)$$

$$= N. \quad (49)$$

Similar evaluation for gD-branes derives the value $-N$. Then the cancellation between tensions can be confirmed as follows. We First we construct a pair of N D-branes and N gD-branes: note that $\Psi_D^{(N)} + \Psi_{gD}^{(N)}$ again becomes a solution since

$$\begin{aligned} \Psi_{D+gD}^{(N)} &= \Psi_D^{(N)} + \Psi_{gD}^{(N)} \\ &= \underbrace{(\Psi_{D+gD}, \Psi_{D+gD}, \dots, \Psi_{D+gD})}_N \end{aligned} \quad (50)$$

where $\Psi_{D+gD} = \Psi_D + \Psi_{gD}$ is the solution for a D-gD pair defined by (16). Then, the tension cancels as

$$\begin{aligned} S_F[\Psi_{D+gD}^{(N)}] &= S_F[\Psi_D^{(N)}] + S_F[\Psi_{gD}^{(N)}] \\ &= +N - N \\ &= 0. \end{aligned} \quad (51)$$

Similar cancellations also hold for other observables.

We next ask whether the cancellation of cohomology occurs between D and gD branes. To see this, let us consider the kinetic operator around $\Psi_{D+gD}^{(N)}$. It is given by

$$Q_{D+gD}^{(N)} \Psi = Q_F \Psi + \{\Psi_{D+gD}^{(N)}, \Psi\}. \quad (52)$$

Right hand side of (52) does not seem to correspond to any known operator with empty cohomology. However, it can be shown that this operator splits into n copies of kinetic operator with empty cohomology for particular choice of Ψ . Such choice of a string field is namely given by a vector of same type as (46) or (47):

$$\Psi^{(N)} = \underbrace{(\Psi_1, \Psi_2, \dots, \Psi_N)}_N. \quad (53)$$

Note that this choice corresponds to a subset of projected string fields considered in [28]. Then, with (42) and (43), it is straightforward to show that

$$Q_{D+gD}^{(N)} \Psi^{(N)} = \underbrace{(Q_{D+gD} \Psi_1, Q_{D+gD} \Psi_2, \dots, Q_{D+gD} \Psi_N)}_N, \quad (54)$$

where Q_{D+gD} is a kinetic operator for a D-gD pair given by (25). Thus the kinetic operator around N D-gD pair splits into Q_{D+gD} which has no cohomology. Therefore, cancellation between cohomology holds for the projected string field (53).

A solution with different numbers of D and gD branes can be constructed similarly. For example, consider

$$\Psi_D^{(N+K)} = (\underbrace{\Psi_D, \Psi_D, \dots, \Psi_D}_N, \underbrace{\Psi_D, \Psi_D, \dots, \Psi_D}_K, \underbrace{0, 0, \dots, 0}_M), \quad (55)$$

$$\Psi_{gD}^{(K+M)} = (\underbrace{0, 0, \dots, 0}_N, \underbrace{\Psi_{gD}, \Psi_{gD}, \dots, \Psi_{gD}}_K, \underbrace{\Psi_{gD}, \Psi_{gD}, \dots, \Psi_{gD}}_M). \quad (56)$$

A sum $\Psi_D^{(N+K)} + \Psi_{gD}^{(K+M)}$ is a solution that represents $N + K$ D-branes and $K + M$ gD-branes. Since $\Psi_D + \Psi_{gD} = \Psi_{D+gD}$, the sum is written as

$$\begin{aligned} \Psi^{(N,K,M)} &= \Psi_D^{(N+K)} + \Psi_{gD}^{(K+M)} \\ &= (\underbrace{\Psi_D, \Psi_D, \dots, \Psi_D}_N, \underbrace{\Psi_{D+gD}, \Psi_{D+gD}, \dots, \Psi_{D+gD}}_K, \underbrace{\Psi_{gD}, \Psi_{gD}, \dots, \Psi_{gD}}_M). \end{aligned} \quad (57)$$

The value of the classical action is given by $N - M$ as expected. However, the cohomology cancel only in k slots in the middle of the vector (57). This can be shown as follows. We introduce a shorthand notation

$$\Phi = (\Phi^{(N)}, \Phi^{(K)}, \Phi^{(M)}) \quad (58)$$

where three components stand for vectors in each N, K and M slots. Then, kinetic operator around $\Psi^{(N,K,M)}$ is evaluated as

$$Q_F \Phi + \{\Psi^{(N,K,M)}, \Phi\} = (Q_B \Phi^{(N)}, Q_{D+gD} \Phi^{(K)}, Q_{gD} \Phi^{(M)}), \quad (59)$$

where $Q_{gD} = Q_F + \{\Psi_{gD}, *\}$ is a kinetic operator for a gD-brane. It is obvious that only Q_{D+gD} has vanishing cohomology therefore cancellation occurs only in the middle K slots. First N and last M slots have no chance to cancel since their Chan-Paton factors do not overlap.

We can further extend above system by introducing off-diagonal Chan-Paton factors of Φ following with the method of [2, 28]. We introduce KMTT projectors $P_k = \Sigma_k \bar{\Sigma}_k$ and arrange them as

$$\{P_0, P_a, P_\alpha, P_A\}, \quad (60)$$

where a, α, A are assigned to three slots of indexes in (57), and $P_0 = 1 - \sum_{k=1}^{N+K+M} \Sigma_k \bar{\Sigma}_k$ is a complementary projector. These labels are identified with Chan-Paton factors for each branes; $0, a, \alpha, A$ are assigned to a tachyon vacuum, N D-branes, K tachyon vacua and M gD-branes respectively. This assignment comes from the fact that a component of string field in each sector obeys appropriate kinetic term in the OSFT action expanded around (57). For example, as for $\Phi_{ab} = P_a \Psi P_b$,

$$Q_F \Phi_{ab} + \{\Psi^{(N,K,M)}, \Phi_{ab}\} = Q_B \Phi_{ab} \quad (61)$$

holds. The component Φ_{ab} corresponds to open string field between D-brane a and b since Q_B is a kinetic operator for a D-brane. A component with mixed indexes, like Φ_{aA} , is interpreted as a string field which connects different kinds of branes.

4 Conclusions

In this paper, we have studied ghost D-branes in the context of open string field theory. First, we constructed the classical solutions for the D-gD system with a help of the MNT solutions. We have confirmed cancellation of gauge invariant observables for the D-gD pair. It have been shown that the BRST cohomology of a D-gD pair cancels. Next we have extended previous result to a system with arbitrary numbers of D and gD branes. We have constructed corresponding solution using modified BCC operators [2]. It have shown that cancellation of cohomology holds for branes with common Chan-Paton factors.

We would like to answer the two questions asked in introduction. First question asks whether gD-branes are physical objects. While we did not find conclusive answer to this, we found that gD-branes belong to a sector of open string field which is not affected by the shift of background. As mentioned in section 2, we can ask whether such sector is gauge away. If so, gD-branes turn out to be unphysical objects and we do not need to worry about negative tension. On the other hand, severe problem will remain if the gD-sector cannot be removed. Multiple gD-branes constructed in 3 make the spectrum of OSFT unbounded below. Therefore, it remains important to give conclusive answer to this question.

Second question asks difference between the original picture [5] of Okuda Takayanagi and OSFT prescription. We have confirmed that the BRST cohomology cancels between D and gD branes. Therefore quantum fluctuations around a D-gD pair cancel expectedly. However, unlike the original picture, the cancellation cannot be extend to the whole partition function. This discrepancy leads rather peculiar phenomenon in OSFT. Let us consider multiple D-gD system. The kinetic operator of a D-gD pair is given by $O_{D+gD} = Q_H$ which is equivalent to that of tachyon vacuum. One can consider an effective action for remaining branes by integrating out string fields for canceled D-gD pairs. As has been conjectured earlier [30, 31, 32, 33], such integration will leave closed string amplitudes with no boundaries, i.e., closed string tadpoles. Therefore, infinitely many D-gD pair implies infinitely many tadpoles. Although this seems rather pathological, useful applications and implications will be expected.

Finally, we present rather positive perspective of our result. Since a D-gD pair is equivalent to the tachyon (or the closed string) vacuum, we can say that the closed string vacuum is described by two vacua in different BCFTs, i.e., one for D-brane and the other for gD-brane. It is expected that an extension to excited states of closed string will leads brand-new formulation of closed string theory in terms of open string. Even if gD-brane is unphysical, it will play a role of auxiliary degrees of freedom which is useful for such formulation.

To push this program forward, we should identify BCFT for gD-brane. Naively, it is

expected that a gD-brane carries same boundary condition as that of a D-brane since boundary states only differ in their sign. In order to identify gD-brane BCFT in OSFT context, one has to derive cohomology of gD-brane. However, direct identification of it seems to be not straightforward, as some attempts presented below indicate. For example, formal homotopy operator of the gD kinetic operator

$$Q_{F+H} = Q_B + \{\Psi_F + \Psi_H, *\}. \quad (62)$$

seems to vanish at least within KBc algebra. This is not similar to the D-brane cohomology whose B/K is a formal homotopy operator. Another but related attempt is finding left and right transformations [19, 34] which connects D and gD-branes. This had already derived by MNT [17] as

$$\begin{aligned} U_L &= MK\left(\frac{F}{1-F^2}\right)cB\left(\frac{1-F^2}{F}\right) \\ &= MKJcBJ^{-1}. \end{aligned}$$

where $J = F/(1-F^2)$. The latter piece $JcBJ^{-1}$ is a star algebra projector therefore has a nontrivial kernel regardless of potential singularity due to $K=0$ poles of M and F^{-1} . Cohomologies of D and gD branes may be related in projected space of string fields obtained by excluding this kernel. To validate this discussion, it is necessary to confirm whether the space obtained by projecting out the kernel is suitable to describe D or gD brane BCFT.

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Conflict of Interest Statement

The author declares that there is no conflict of interest regarding the publication of this paper.

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